UCRL- 89674
PREPRINT

Fabrication of Machined Optics for Precision Applications

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Instrumentation Engineers
Kongresshaus Garmisch-Partenkirchen
West Germany
04/12/83 - 04/14/83

August 02, 1983

Lawrence Livermore Laboratory

FABRICATION OF MACHINED OPTICS FOR PRECISION APPLICATIONS

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Introduction

This paper will discuss the current state of optical fabrication employing precision machining. While it is a perspective based upon both our own work and that of others, it lays no claim to being a comprehensive survey.

Today state-of-the-art precision machining can produce surfaces to tolerances consistent with visible and near-visible wavelength optical applications. These new technologies extend not only the range of geometries, but the range of materials, and in some cases, even the range of finishes available to the optical designer. These technologies are not an abrupt step increase in capability but the result of several decades of advances in metrology, computer technology, servomechanisms, bearing technology, tooling and environmental control.

The most precise work today is done with single point diamond tooling, but we find the commonly used term, "single point diamond turning" (SPDT) an increasingly inadequate descriptor of the synergistic union of technologies that is the precision machining of today. With the entrance of cubic boron nitride (CBN) into the field, it is no longer the exclusive domain of diamond. The same grouping of technologies is being applied to grinding on a small scale, so it is no longer the restricted domain of single point tools. Yoshioka et al., for example, have reported 0.2 micrometer resolution on a new surface grinder which has produced plastic regime grinding on brittle materials such as fused silica[1]. Finally, even on turning machines, we are increasingly seeing flycutting and occasionally, on small work, even dual spindle flycutting of flats and spheres, so it is no longer just turning

With third generation machines coming on line, precision optical machining is no longer a novelty, but is entering the realm of a maturing technology. As metrology techniques advance to a precision of a few atom layers, those gains due primarily to metrology are slowing down and future gains will be seen to result from careful design, attention to increasingly miniscule detail and to tooling and materials studies. With maturity, the field is less hampered by the claims of the overenthusiastic and the fears of the ignorant. It is seen less as a competition to the optical industry than as a process competing for their attention; able to extend their capabilities and productivity. While some components can be carried to completion, many more require the attention of the traditional optical finisher. To give a feel for the state and pace of the field, this paper will outline the development of the technology, compare its advantages and limitations with traditional methods of surface generation, describe a few of its more novel applications, discuss some elements of machine design and necessary approaches to metrology and finishing.

Historical perspective

During the decade 1955-1965, and prior to an interest in diamond machining, the engineering discipline now known as "precision engineering" was developed extensively within the Lawrence Livermore National Laboratory and other agencies of the U. S. Atomic Energy Commission. Initially, this work focused on dimensional metrology applied to coordinate measuring machines, studying factors such as positioning accuracy, straightness of travel and angular motions of linear axes, analogous parameters for rotary axes, and mutual squareness or parallelism between axes. It was found that these errors of machine geometry were often quite small in comparison to the total machine error, and that the dominant factor was usually non-repeatability due to transient thermal distortion from external or internal heat sources[2]. By systematic testing, the various error sources could be isolated and reduced, resulting in major improvements in overall accuracy.

Since machine tools are fundamentally similar in design to coordinate measuring machines, the same techniques were subsequently applied to improving the accuracy with which workpieces were made.[3-6] This situation was more difficult due to additional error sources from deflections due to the cutting force, heat produced during cutting, wear of the cutting tool and variability of cut depth from the built-up edge on the tool[7-13].

Given this background, the promise offered by the diamond tool was immediately evident[5]. With its ability to make mirror-finish cuts less than 0.1μm in depth, resulting in negligible heat and tool force, plus its remarkably low rate of wear, the level of machining accuracy could immediately be advanced to that of the finest measuring machine. Indeed, the first diamond lathe built at this Laboratory during 1969-71, Diamond Turning Machine No. 1 (DTM-1), was a modified commercial measuring machine[14,15]. (Over a dozen diamond lathes now operating at various installations are basically similar in design).

While the first uses of diamond tools date back over a century, there are good reasons why the use of diamond machining for optical applications has only occurred within the past decade. First, the necessary knowledge of precision engineering outlined above had not been developed at an earlier date. Secondly, the optical surfaces that could be generated by motion of a single rotary or linear axis, such as spheres, flats, cylinders and cones, were either readily produced by conventional optical fabrication or of little interest to optical designers. To create general conics and other aspherics requires the coordinated motion of two or more machine axes by a numerical control system, and hence needs the support of modern digital electronics and computers. Also, the measurement of the axis travels to adequate accuracy for machining optics was difficult if not impossible prior to the availability of long-travel laser interferometers.

Initial uses of diamond turning at this Laboratory and other government installations were not primarily for optical purposes. The first significant optical attempt, a fast parabola of 15 cm aperture and 3.8 cm focal length, was made in 1972 on the original machine with a limited-capability numerical controller driving one rotary and one linear axis via leadscrews. The pure silver surface had a reflectivity of ~98% at 633 nm and interestingly retained a tarnish-free appearance in open atmosphere for over a year. However, interferometric testing showed significant surface waviness, predominantly from periodic errors in the rotary table worm drive system, repeating every 2° and yielding ± 15 sec of surface slope error. The undesirable result was a line focus extending some 25 μm along the optic axis[16].

By the late 1970's, the diamond machining technology had progressed to the point of being able to fabricate aspherics of decent optical quality, although the emphasis still was not on optics and the technology remained primarily in the former AEC (now Department of Energy) installations. The U.S. Air Force, based on its need for production quantities of heat-seeking missile scanner mirrors of 25 cm aperture operating in the 8-12 μm region, sponsored an effort here to transfer the necessary technology to the private sector[17]. This effort was successful in helping to create a diamond machining vendor base for the sponsor as well as commercial sources of complete systems for diamond turning or flycutting[18-22]. These systems are capable of figure accuracies in the range of 1-2 μm for 30 cm apertures, improving for smaller sizes, and surface roughnesses that are quite adequate in the IR. As a further benefit, use of commercial diamond machines has been stimulated in other optical applications, such as Fresnel, opthalmic and other plastic lenses, either directly or in machining injection molds.

As indicated, the first generation of diamond turning machines were modified measuring machines. With the experience gained on these, a second generation of machines was specifically designed for this purpose. During the past decade, a number of manufacturers have begun to establish product lines, but they are frequently tailored to special purposes by the joint efforts of manufacturer and user, making it difficult to separate them into categories. The experience gained in their design and operation has contributed to the science of machine design to an extent that it seems clear we are seeing a third generation of machines today, but the advances have been so continuous that it is difficult to identify a transition point.

At LLNL, we are in the process of bringing three third generation machines on line and upgrading a fourth machine, a 20-year old ExCello. The three new machines are a large two-meter swing horizontal lathe, DTM-3, by the Bryan group; a 1.6 meter vertical lathe, the Large Optics Diamond Turning Machine (LODTM), by the Donaldson group; and a small 10 cm horizontal Precision Engineering Research Lathe (PERL), by the Thompson group[23,24].

As indicated, LLNL personnel have been involved in these developments from their inception, and in-house experimental designs have figured largely in our literature since these allow new approaches to be tried. However, it should not be inferred from this historical sequence that we recommend an in-house design as a first effort today. We do own and operate commercially built equipment, and as a group, the DOE installations, of which we are a part, are major purchasers of such equipment.

Fabrication advantages of machining

All precision machines are, to some degree, measuring engines. The newest machines can resolve incremental displacements between slide and built-in reference as small as 6A, and absolute position to a few parts in 10^8 even on larger machines. These systems are global and absolute as compared to the differential measurements used in traditional optical fabrication. They are continuous during the process as compared to the intermittent testing in the traditional manner. Furthermore, a machine usually repeats itself to a degree more precise than its absolute machining accuracy, so it can be re-programmed in a cut-measure-correct-cut mode to the limit of its repeatability, if a more accurate figure measuring instrument is available, regardless of the shape cut. Traditional optical fabrications have the added advantage of a self-correcting mode in the case of spheres and their limiting case, the flat; however, this self-correction toward the spherical also limits geometries to spheres and near spheres.

The range of machined geometries allows simultaneous machining of reference surfaces for testing, alignment, and assembly. The process is repeatable and absolute, well suited to production and precision assembly. In conventional spherical optical fabrication, figure and radius are seldom achieved to equal degrees of precision. In aspherics, conventional techniques laboriously wear a shape closer and closer to the desired class of mathematical figures and often further and further from the desired particular figure of that class.

Machined surfaces tend to have very high specular reflectivity, particularly in the infrared, and very high-pulsed laser damage thresholds[25]. Indeed, these are close to the "intrinsic" or theoretical values for the material. When scattering due to turning is unobjectionable, the surface may be finished with the turning operation.

On the debit side of the ledger, the technology is sophisticated and costly, from the design of the machine to its environmental siting, operation and maintenance. The cost frequently extends to chucking, tooling, programming and premachining preparations[26].

It is, at present, restricted to a small range of materials, usually metals that do not form carbides (aluminum and some forms of nickel appear to be exceptions to this rule). The surfaces are usually rougher in the directions of the tool feed per revolution than those conventionally polished, although some small flats and very long radius spheres are now being turned below 20A rms[27].

Optical applications of precision machining

The most obvious application of precision machining is metal reflectors[28]. The difficulties encountered in the fabrication of these by traditional methods are illustrated by the fact that conventionally fabricated mirrors are almost invariably metallized surfaces on polished glass. However, there are many potential applications for metal mirrors such as integrally cooled mirrors, pulsed laser reflectors, lightweight structures, and irregular shaped mirrors that warrant machining even for simple flat and spherical surfaces today. As the cost of machines and the waiting lists for the better machines go down and surface quality increases, this technology may capture an increasing share of this market.

Figure 1. Early group of aluminum off axis paraboloids. Interrupted cuts are not perceptibly inferior to continuous cuts.

Figure 2. Early X-ray grazing incidence telescope. All X-ray optics require subsequent polishing.

We have seen several cases where simple flats required the special capabilities of turning. One case was the copper mirrors for the Los Alamos Antares program where the superior laser damage threshold of as-turned surfaces was required. A second case was the 27 cm KDP windows for the Livermore Laser Fusion Program. Here, there was a combination of requirements. An array of nine segments was required with thicknesses matched to a fraction of a micrometer, allowing no edge roll off, with extremely tight crystal axis orientation and edge squareness, etc. This task demanded jigging to optical tolerances and a state-of-the-art flycutting operation. Less obvious were the demands of the surface. KDP is extremely awkward to polish. Slight traces of water permit residues to recrystalize providing crystalites growing with different hardness along different axes. These are serious sources of scratches. While conventional polishing of these large crystals was not attempted, the prognosis for clean polished surfaces in this size was quite poor.

Fresnel lens technology was one of the first beneficiaries of precision machining, progressing from dies to actual turned optics, and finally to undercut precision fresnels. In the latter case, the undercutting, possible in no other way, greatly reduces the ring effect which can be quite objectionable for some applications.

It is in metal aspherics, however, that turning really offers completely unique capabilities. Among the most extreme cases are the x-ray grazing incidence optics, particularly those of the Wolter and Schwarzschild types. These near-tubular optics are so difficult that they are seldom attempted any other way. Of particular note are the 3 to 5 cm aperture Wolter Type I X-ray microscopes turned by the Bryan group on DTM-2[29]. Turned within a 2 microinch band, these optics are a definite milestone in precision machining, demonstrating that machines with diamond tools are capable of working to this level. These also demonstrate that turning may be but one of a series of optical fabrication processes. Even at this precision, these objectives were nearly an order of magnitude away from the figure specifications and two full orders away from the required smoothness. Turning thus was but the generation process preceeding a conventional polishing operation, but what a generation process!

For X-ray telescopes, the turning process is not only capable of generating the rough shapes to a precision unobtainable by other means, but capable of generating these to the absolute accuracy necessary for nesting. Equally important, it is capable of providing precision assembly joints and perhaps most important of all, alignment reference surfaces for subsequent optical testing[30].

Other classes of metal aspherics virtually unobtainable in any other way are laser resonator optics such as axicons, waxicons, and reflaxicons -- optics having non-zero slope on the optic centerline[31]. While most of this unstable resonator work is at IR wavelengths, these are often much more sensitive to figure errors and surface roughness than normal IR optics. Multiple passes make them extremely sensitive to near angle scatter and hence long spatial wavelength errors; figure requirements of $\lambda/20$ to $\lambda/100$ in the visible are not uncommon. Furthermore, those designs that expand the compact area near the center to an outer ring are extremely sensitive to azimuthal errors since recompaction can produce severe phase distortion around the compacted centerline.

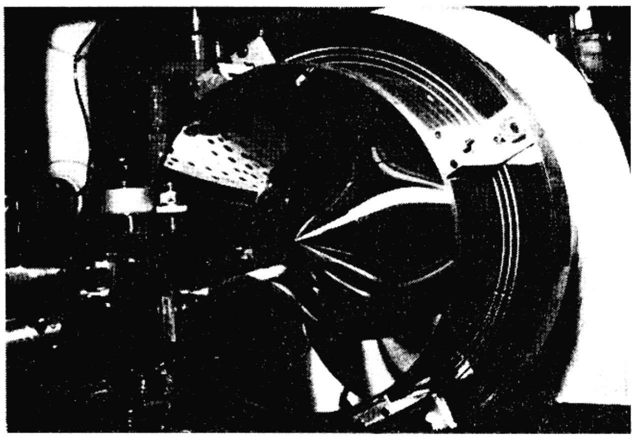

Figure 3. Early unstable resonator linear waxicon, turned in one piece on oil showered 1st generation machine. Single piece turnings of this type are virtually impossible to polish, but linear surfaces from 3rd generation machines may not need polishing.

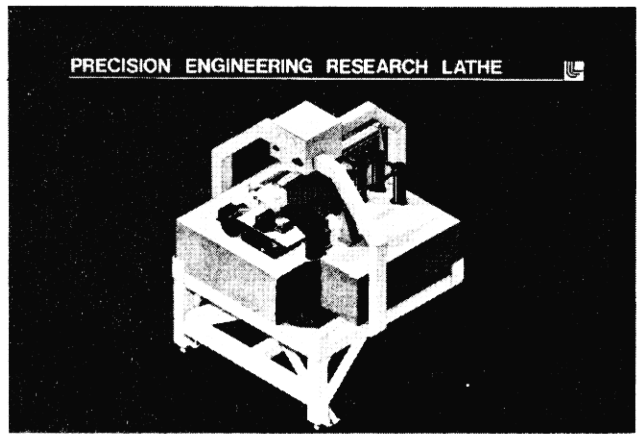

Figure 4. PERL, a 10 cm 3rd generation machine for tooling and machining science research. Small size increases rigidity and reduces external thermal, accoustic, and seismic coupling.

Machine design

We have made the point that diamond turning is simply the logical extension of precision machining. A diamond turning machine is a lathe which can use diamond tools to improve accuracy or finish. Diamond tools can and have in fact been used successfully on a number of conventional machines. It is obvious that not all of these are appropriate for optical fabrication.

The design of successful machines varies widely. Single axis machines used in either turning or flycutting modes are sufficient for flats and, by tilting the head, for cones or cylinders. Most are two and three axis machines sometimes combining rotary with one or more linear axes. Most machines have horizontal spindle axes, but a few large machines have been built with vertical axes[24,32]. There is thus no single approach to design.

What is common to all successful designs, however, is the degree to which designers have addressed in detail potential sources of error in the machine and machining process. Errors normally considered insignificant must be considered. For example, the size change in large components due to variations in barometric pressure is often significant.

Careful attention is paid to making the structure as stiff as possible and to raising the natural frequencies. Stiffness aids in reducing fluctuations in cut depth due to disturbances such as variation in material thickness to be removed or material hardness. There is evidence that stiffness affects machinability; for example the 10 cm BODTM, forerunner of the 10 cm PERL machine has been able to machine materials that larger, softer, machines have not[33]. High natural frequency allows a wider bandwidth in the axis servo controls and hence better rejection of the various disturbances.

While many different slide configurations are used, a few elements of uniformity are emerging. On the newer machines, slides have hydrostatic bearings, usually oil to take advantage of its damping properties. Lead screws are being replaced with capstan drives. Another element of uniformity is direct coupling of motors to spindles. This insures that motor perturbations as a function of rotational position do not contribute to surface roughness and are possibly correctable for their influence on figure.

Another element of machine design frequently overlooked is ruggedness. Parts of the precision necessary are long lead items, and machines do run away and crash. The ruggedly elegant simplicity of the bare granite surfaces of DTM-3 is testimony to long experience.

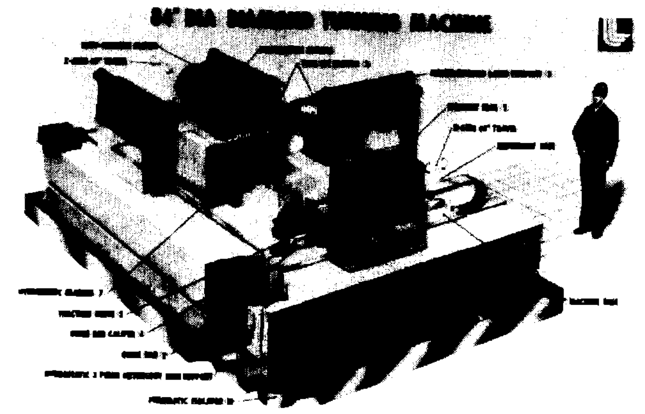

Figure 5. DTM-3, a massive 2.1 m 3rd generation machine. Ruggedness and precision can be compatible.

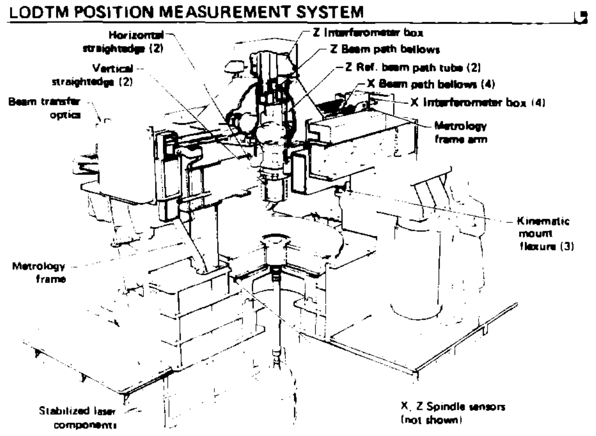

Figure 6. LODTM, a vertical spindle, 1.6 meter 3rd generation machine designed for quarter wave visible spectrum optics. The metrology frame isolated by $\pm$ 0.0005° C water is the largest super invar structure to date.

Chucking

Chucking is a continuing problem frequently unique to each part, and sometimes to each workpiece. In many cases it requires careful planning from a team that includes the optical designer, machine technologists, polishers, metrologists, and users. In some cases it is finally machined both immediately prior to part machining and during part machining to insure mounting trueness and to incorporate as-machined references. In the most successful operations the chucking has followed the part through all of the fabrication processes and it can continue its support role even as part of the final assembly.

Chucking must insure that the part does not deform during or after machining due to such forces as gravity, turning accelerations, or its own coupling either to part or machine. On this scale of tolerance, the addition of stresses and particularly their removal during machining are a problem. Iterative chucking and adjustment to conform to the new shape resulting from these changes may be necessary between successive cuts and operations, especially on thin tubular parts. Even plating between operations can introduce stresses sufficient to distort the shape.

Sometimes chucking and turning operations must take place in an environment continuously controlled to a fraction of a degree from inception of chucking through final machining. The laser fusion 27 cm KDP crystals turned on DTM-1 required such treatment for several reasons.

Hard, kinematically determinate coupling between part and chuck is usually preferable, but not always possible. Some parts are so weak they cannot be accommodated by such.

Sometimes chucks are purposely built to produce an intentional deformation. Y-12 engineers, for example, have demonstrated chucks with reverse lips mirroring those of the part to provide a deflection at the chuck face equal and opposite to that which would otherwise result on the part from centrifugal forces, reducing edge displacements considerably.

Machine metrology

All precision machines are to some extent measuring engines, and all rely on some form of feedback whether via the operator or by closed loop servo. The primary purpose of the metrology is to determine the position of the tool with respect to the workpiece. On conventional machine tools this metrology is usually a rotary encoder or resolver attached to the leadscrew to determine the position in the direction of travel, assuming adequate slide straightness to ignore movements in the orthogonal directions. The second generation of machines usually employ commercial laser interferometers in the direction of travel only, and improved machine geometry. (The application of the interferometer to machine tools was first made by Michelson near the turn of the century, although this did not become practical until the invention of the laser.)

The current commercial interferometers are accurate to about 3 parts in 10^6 in air and somewhere in the 10^8 range in vacuum. This is no longer adequate for large state-of-the-art machines which require on the order of one part in 10^9. For these machines, iodine stabilized helium neon lasers operating over most of their path in vacuum or enclosed helium filled tubes are yielding the required performance. Increments of displacement in the direction of measurement today can be resolved to about 6A, about two atom layers. This however is not the absolute accuracy of the interferometer. Polarization mixing, stress birefringence, etc. combine to reduce absolute accuracy on a large machine to about 25A per interferometer[24].

A key element for large third-generation machines is the metrology frame, a separate subframe that is used as a common reference structure for all geometry measurements. The metrology frame must be kinematically mounted, temperature-controlled and kept free of any variable forces that would alter its size or shape. (This concept was introducted to diamond turning machines by Bryan[23], but appears to have first been employed by Zeiss on a measuring instrument in the 18th century).

While laser interferometers can provide adequate distance measurements, other errors such as slide straightness, squareness, spindle motion and spindle growth must also be considered for third-generation machines. Typically these are detected with short-range high-resolution sensors reading reference surfaces such as straightedges, with all measurements being made with respect to the metrology frame. Reference surface errors are handled either by fabricating the components to acceptably tight tolerances or by storing calibration data in look-up error tables in the control computer.

Environmental control

Even with the most sophisticated available metrology hardware of the preceding section, poor figure and finish can still result from two disparate error categories. One is drift, i.e., a change in size or shape of structures that support and interconnect the interferometer optics, gageheads, reference surfaces, etc. Here, thermal distortion due to temperature variation is the major culprit. The second category is vibration, i.e., tool-to-work motion at a frequency beyond the bandwidth of the metrology and closed loop axis servo error-correcting systems. Because of the remarkable cutting fidelity of the diamond tool, such motion is faithfully reproduced in the form of increased surface roughness.

There are several approaches to temperature control. Obvious improvement comes from eliminating or reducing large and/or variable sources where possible, or at least removing them from the machine structure. Ambient air temperature is a disturbance source that can be reduced by closer temperature regulation, but air flow over the machine is relatively poor for control of machine temperature against internal heat sources because of its low heat capacity and heat transfer coefficient. Liquids such as oil or water are much more effective. We have used both in a variety of forms, ranging from open flows over an entire machine structure to closed flows near a variable internal source such as a spindle bearing[13,15]. Temperature control of sizeable flows (50-100 gpm) can be accomplished rather simply with ordinary heat exchangers rejecting heat to chilled water, with a feedback scheme that varies the chilled water flow rate as the basic control mechanism. Simple on-off control of the chilled water flow has yielded $\pm$ 0.01°F performance, and continuous variation with proportional-integral-derivative control has extended thus to $\pm$ 0.001°F. The latter approach has also been used in an air-to-chilled water coil to maintain 20,000 cfm of recirculating air to $\pm$ 0.01°F. Thermistors are used as sensing elements because they have sensitivity to 10^{-4}°F or better, and can be quite stable in the glass-in-bead form if properly pre-conditioned.

The use of low-expansion materials can be an aid in controlling drift due to temperature variation. However, care must be exercised, since if the workpiece material has a larger coefficient of thermal expansion than the machine structure, thermally induced errors can easily be larger than if the entire system had the same larger coefficient.

Another source of difficulty is secular drift or dimensional instability, in which the microstructure of the material is not in equilibrium, and may change size and shape over a period of many years. Since good measurements are difficult and time-consuming, information on a given material tends to range from sparse to nonexistent.

Vibration control begins with the machine tool itself, where any audible noise source is apt to be a problem, from the hum of electric motors down to the exhaust hiss of air bearings. The immediate environs must also be considered. Normal shop noise is unacceptable, and third-generation machines are located in sound-deadened rooms in which conversation is prohibited during a finish cut[34]. Seismic isolation is also required, typically by use of self-levelling pneumatic isolators.

Machine system accuracy

While the accuracy of individual components of a precision machine tool can be held to very small values, such as the 25A accuracy of individual laser interferometers mentioned previously, it is also true that there are a large number of different sources of error from metrology, chucking, environment, etc., when one is working at such a fine level. The combination of all these errors can be approached through a formal error-budget process similar to that used for complex optical systems. This approach has been used for LODTM, yielding an estimate of 1.1 μm rms (about $\lambda/20$ rms in the visible), or about $\lambda/4$ peak-to-valley for figure accuracy in a 1.6m aperture[35].

Mechanical part metrology

While optical testing such as interferometry is an obvious first choice for an optical component, it runs afoul of the same difficulty as conventional optical fabrication does when the component is strongly aspheric and afocal; making a null element may be as difficult as making the original optic, and one has no definitive way of optically testing either one separately. In such cases the same technology that is employed in diamond machining can be attractive in measuring it.

Use of the same machine tool to machine a part and then measure it, with a suitable gagehead replacing the tool, has an inherent pitfall that must be avoided. Since the machine is going through the same motions, any and all repeatable errors of the machine will not be detected during the measurement pass. However, in given situations, it may be possible to qualify the machine to an adequate degree in the work zone of interest by using it to either machine or trace another optical component, such as a flat or sphere, that can be tested interferometrically.

There is at least one example of part metrology in which mechanical testing is at least as accurate as interferometric testing, even for spherical optics. This is testing for roundness, or more generally, for symmetry of shape around a central axis. Mechanical roundness measuring instruments, based on aerostatic spindles, have been demonstrated to absolute accuracies of 25A peak-to-valley, or $\lambda/200$ in the visible. While the spindle errors are not of this level, typically being a factor of ten higher, they are highly repeatable, and hence can be subtracted from the measurement once they are known. Separation of repeatable spindle error from part error is readily accomplished by a reversal technique that is the conceptual equivalent of the classic three-straightedge intercomparison test[6]. Instruments of the above accuracy are necessarily equipped with microcomputers for data acquisition and reduction, which also allow for multiple-measurement averaging, spectral decomposition and to the like[24].

Surface finishing

The primary difference between the machining of optics and that of other parts generated to similar dimensional precision lies in the consideration given to the surface. In optics there is a functional effect of surface structures down to sizes somewhat below the wavelength of interest and from subsurface structure to a similar depth. Decker et al. have pointed to evidence of remarkable crystalinity of diamond turned surfaces (i.e. lack of amorphous smearing) and attributed to this the high, almost "intrinsic" reflectivities and pulsed laser damage thresholds obtained by simple turning[25,27,36]. Since the depth of subsurface rearrangement is a function of shearing forces and the stresses generated along the cutting edge, it is not unexpected that the surfaces generated by shallow cuts with low rake angle, micro sharp, long-wearing tools free of built up edges should show some difference when compared to those produced by conventional tools. The diamond turned surfaces may show up to an order of magnitude higher damage threshold than conventional polished surfaces and even the most sophisticated polishing techniques can seldom provide levels more than 80% of these values. We also see what appears to be lower rates of tarnishing and surface oxidation on as-turned surfaces in air storage. There is little visible effect on silver after a year of storage and machined aluminum can appear freshly turned after several years of storage. At least one study using accelerated chemical tests has been unable to detect chemical differences, but we are confronted with historical evidence that one may exist.

Note that we are here addressing both surface structure effects and substructure. These result from a careful sequence of cutting depths. A final extra fine cut cannot restructure subsurface strain resulting from an immediately preceding coarse cut.

While the surface and near surface crystaline structure resulting from careful turning is superior to that obtainable by conventional techniques, the periodic groove structure and low amplitude ripple from lead screws, machine dynamics etc., are not. Groove spacing ranges from several to several tens of micrometers and a warped lead screw can produce periodicities of up to three millimeters. These have amplitudes ranging from 0.01 to 0.5 micrometers.

To a first approximation, the grooves mirror the tool shape, precisely mirroring minute tool defects. However, because of machine vibration from various perturbations, adjacent grooves may not lie at the same height. The outer edges of the grooves rise slightly higher than simple geometry predicts and chip scorings are on the order of tool defect irregularities. Finally, as machine errors are brought under control and metrology advances, other fine structure due to workpiece tool interactions is appearing.

These surfaces obviously will scatter light differently from polished surfaces, directing it primarily normal to the grooves. Scattering from turned surfaces has been studied by a number of authors[36-47]. Polished surfaces with spatial frequencies below 2 micrometers dominating, yield wide angle scattering predominantly at angles above 10 to 15 degrees. Turned surfaces from second-generation machines tend to scatter at angles within a 3 to 5 degree cone.

The amplitude of the surface finish irregularities is quite adequate for most infrared work, and increasingly in small flats and shallow spheres, for near visible and even visible wavelength applications. However, even in the infrared, there are astronomical applications where signal strengths orders of magnitude below noise demand surfaces rivaling X-ray optics for sophisticated recovery techniques. The periodicity and low angle scattering are very serious problems for resonator work even in the infrared. The surfaces are completely inadequate for uv and X-ray grazing incidence work that is of increasing importance in astronomy, fusion diagnostics, and synchrotron radiation studies. However, the figure obtained is much more frequently adequate or near adequate, and very often unobtainable in any other way.

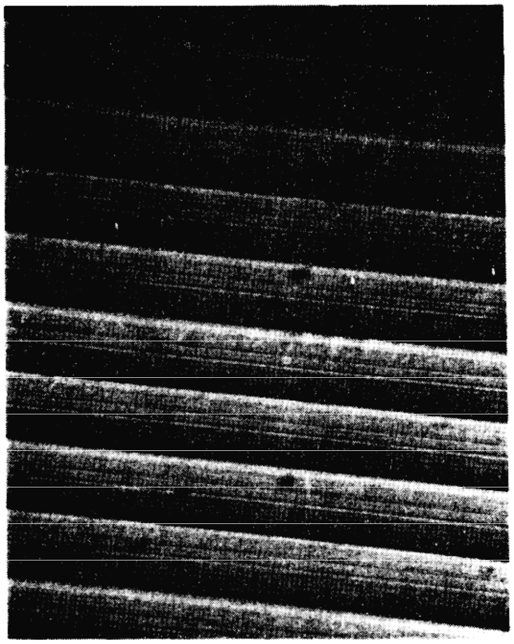

Figure 7. Nomarski micrograph of 2 microinch P-V cut at 30 micrometers feed per revolution. Coarse cut selected for illustration. Approximately 600:1 height to width aspect ratio.

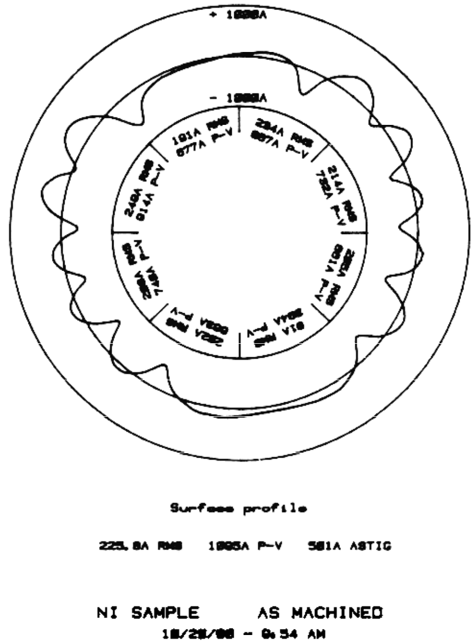

Figure 8. Optical heterodyne profiler 200 micrometer diameter circular scan of Ni surface shown in Figure 7. Scan is distorted since instrument attempts to remove biaxial symmetry as astigmatism.

There are trade-offs that face optical designer, machinist, and polisher in these latter cases. Polishing must to some extent be transverse to the grooves. As can be inferred from Appendix B, polishing of surfaces convex with respect to the direction of polishing motion is much simpler than for concave surfaces. Tension alone can stretch flexible belts and membranes across the grooves from peak to peak. A concave surface however requires a pressure that may force a polishing pad, sufficiently flexible to follow contours, down into the groove troughs to polish these away almost as rapidly as the peaks, propagating the irregularities through the surface. This is shape and frequency dependent, at the control of designer and machinist with abrupt transitions from feasibility to infeasibility[48].

To a first approximation, groove heights vary inversely with tool nose radius and as the square of the feed per revolution. On the other hand, tool loading increases with tool radius, placing greater demands on setup rigidity. Increasing radius places more demands on machine stiffness and can lead to chatter. Decreasing tool feed extends machining time, increasing the probability of environmental transients and possible effects of tool wear.

The approach to the polishing operations is hardly ordinary. The figure of the machined part is frequently within a fraction of a visible wavelength of the finished optic. It thus begins its course through the shop in what would normally be termed the final stages. However, the shape is seldom that of a conventional optic, and the specifications for smoothness and figure are frequently more severe than normal. Specifications for less than 5A rms smoothness at frequencies greater than 10 cycles/mm and peak-to-valley deviations from ideal as low as 50A have been requested. To date, these probably have not met on aspheric surfaces, but they have been met on small flats and spheres. In the latter cases, they are, in fact, being approached by as-machined surfaces and these tolerances do not seem impossible for small X-ray microscopes of the Wolter type.

Precision polishing follows the same path as precision machining. The primary emphasis is on metrology. When adequate metrology is available, the mechanics of the operation must be examined, understood, and applied in a rational fashion. For X-ray optics, detector pixel size is no smaller than visual wavelength spot size, so, at the present time, the mean figure generated by state-of-the-art machining is frequently adequate. The emphasis from a metrology standpoint is then on submillimeter period smoothness.

There are several approaches to surface metrology, each unfortunately limited to small flat samples. One is through stylus measurement. At the present time these instruments can resolve to near tenth micrometer period and about 2 angstrom amplitude[49]. Heterodyne instruments employing the Zeeman split frequencies with differences stabilized to master crystal oscillators are resolving surface features to about 2.5 micrometers period and amplitudes on the order of 1 to 2 angstrom. Both the limit to date of the heterodyne instrument and glass polishing technology are shown in Figure 11, an 0.2 mm circular traverse (512 data points) on a piece of polished fused silica with readings taken ten minutes apart. The fundamental (sin θ) representing tilt and (sin 2θ) astigmatism terms have been filtered out mathematically. This instrument is bandlimited to between 5 and 400 cycles/mm[50].

Figures 7 and 8 show an as-turned flat specimen from a simple turning. This is not representative of the current state of fly cutting with longer (2.5 mm) radius tools. Figure 10 represents the current limit of polishing electroless nickel flats (5 cm dia) on a unidirectional planetary lap. We typically find the roughness of stroked laps at the same conditions to be 1.5 times those found on unidirectional laps. At the present time, we do not have adequate metrology for aspheric surfaces but indirect readings infer another factor of 1.5 to 2.0. For example, we have produced one 400 mm dia tubular x-ray telescope with a measured scatter consistent with 22A rms. This optic was polished only through 1/2 micrometer diamond which normally yields 5.5 to 9.0A rms on unidirectionally polished 5 cm dia flats. The optic was gold-coated after polishing and before testing, a process that usually increases the surface roughness slightly. In addition, this instrument was polished with an oscillating stroke adding a factor of 1.5 and leaving a factor of 1.5 unaccounted for, presumably due to aspheric induced flexing.

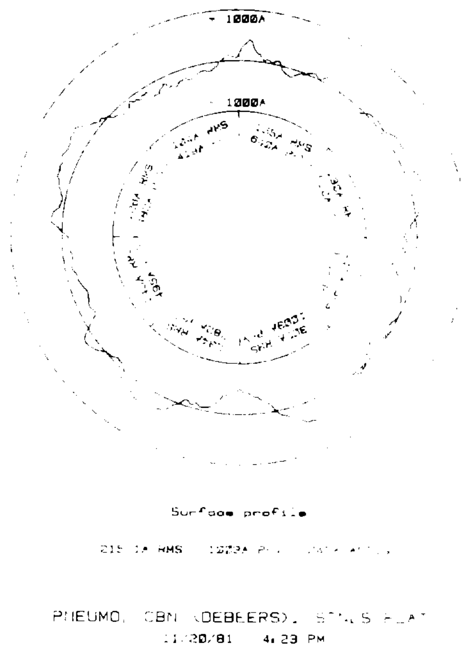

Figure 9. Optical heterodyne profiler scan of stainless flat cut by composite CBN tool. Readings as low as 100A rms have been obtained on small samples.

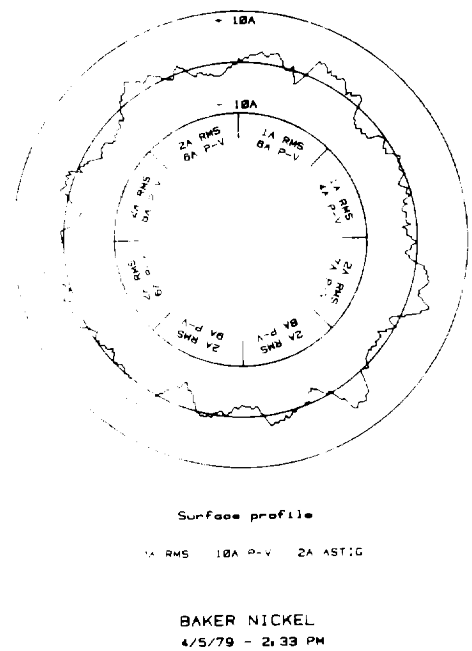

Figure 10. Scan of 5 cm unidirectionally polished Ni specimen. 2.0A rms reading is 1.2 to 1.5 times lower than average for this process.

Although smoothness may be emphasized over figure functionally for some applications, figure, smoothness, and cosmetics such as scratches and digs are operationally not as separable as they are semantically. They all represent geometric deviations from some smoothly varying surface differing only in the range and amplitude of spatial frequency components. Poor cosmetics, for example, are an indication of poor lap fitting allowing at some point ingress of larger particles or agglomerates. Similarly, poor fitting permits separation and rearrangement of abrasive particles on the lap degrading smoothness. Thus extremely tight specifications on any one of these three may require much closer attention to the other two than their specifications apparently require.

The smoothest surfaces usually require continuous contact between lap and part to avoid an edge scraping abrasive loose for deeper scoring than the mean. Furthermore, near uniform wear to maintain the mean figure provided by the machining process usually requires this same continuous contact in plane translation over the area of uniform wear. Since stroking is required, the areas subject to intermittent contact may not only be less smooth, but less equally worn. For critical applications, these areas should be outside the clear apperture, obscured by baffeling or removed between polishing and assembly. These may be movable surrounds or extensions, pinned to the primary optic and machined with it as continuations of the curve, or in some cases material to be machined or etched away in a post polishing operation.

Just as an understanding of tool mechanics is required for precision machining, so it is in polishing. It appeared that the process was not simply scalable, so a mathematical modeling approach was taken. Nomarski and SEM examinations indicated a gouging model was required for metal polishing. Further, just as peening-spallation appeared to be less effective in machining than fixed tool shearing, so a fixed charge polishing procedure was selected with pitch holding the abrasive in near fixed orientation (i.e., in lieu of a tool holder) rather than using continuous addition of free rolling abrasive. Since most machined optics had been formed with diamond and since this material was available more finely and closely graded than most others, this was selected. Diamond is as hydrophobic as any material known so oil was selected as a lubricating vehicle. Since the fixed charge would slowly sink into the pitch reducing part-to-lap gaps much below those found in conventional free slurry polishing, very low viscosity was required. Silicone oils are quite neutral and tenacious and could aid in retarding oxidation and tarnishing so these were selected.

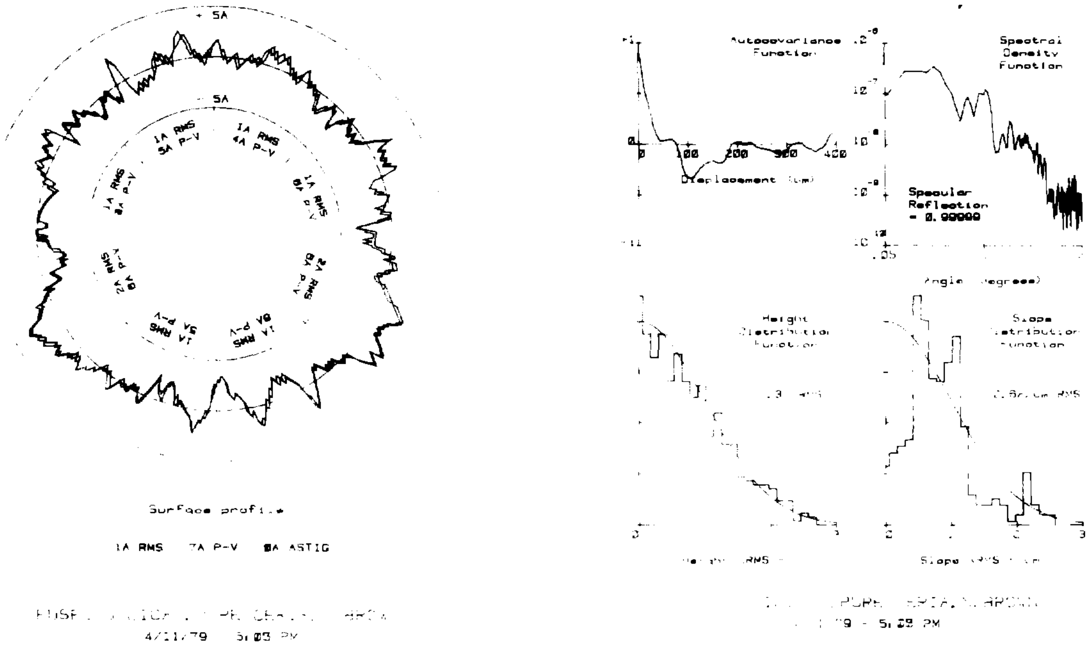

Figure 11. Double scan, 10 minutes apart, on fused silica specimen. A comparison with Figure 10 shows metal polishing technology does not lag far behind glass polishing.

Since the constant or near constant function in the Preston wear equation required units of inverse pressure, both elasticity and hardness measured in load per unit of indentation were obvious candidates. Most available data on polishing favored hardness as the dominant effect, but early polishing experiments on very soft copper and much harder electroless nickel in this superpolishing regime showed polishing rates differing on the order of 2 for identical conditions[51] Since this is the order of their Young's moduli, an elastic model was selected.

A very simple model based on spherical particles for dimensional consistency yielded a constant for the wear equation that is the inverse of twice Young's modulus. Data for electroless nickel polished in the fashion indicated shows constants clustering about the predicted value.

The model included particulate size and concentration in the development but these cancelled out. The model was consistent in predicting strains below or very near the elastic limit. The model also predicts gouging depths in terms of particle diameter, pressure, concentration, and Young's modulus. However, the gouging depth predicted for constant size particulates is meaningless since it shows penetrations of less than an atomic dimension.

Particulate size distributions were measured and found to be gaussian cumulative distributions when plotted as weight fractions as functions of the log of particle size. For diamond, the standard deviation was found to be near the log of $\sqrt{2}$ and for other materials near the log of 2. Pitch penetration studies using balls provided a load, viscosity, velocity, size relationship dimensionally similar to Stokes law. This permitted the pitch to be used in a manner analogous to a load cell to examine penetration distributions and to determine the parametric influences on fixed charge lap life and unit area polishing removal capacity.

The estimations of polishing rate, roughness, and removal capacity are especially important in polishing machined optics, many of which are plated. In some of these, turning, polishing, and figuring take place in thickness as small as 10 micrometers, but most are on the order of 50 to 70. Even so, there is little room for experimentation. The polisher must know rates to predict polishing off a small hump. He must also estimate removal capacity. In a fixed charge pitch lap, the maximum smoothness is achieved just as the full charge distribution is contacted and just about to pass below the surface. A fresh charged lap sized too large will not reach this point when it has removed the proper amount of material. One too small will score deeply in its initial cut, possibly to a depth it cannot reach in removing its entire capacity when averaged over the workpiece.

The observation that rate should be independent of size to at least first order while roughness is size-dependent predicts smoother polishing with little sacrifice in speed by going to finer sizes.

The model has not been extensively tested but appears to work well with electroless nickel and reasonably well with non-carbide forming materials when used with diamond.

There are discrepancies when the model is applied to alumina abrasives. Further work has shown that these have a tendency to move about on the lap. Alumina is hydrophylic and in the presence of water does not wet well with pitch. Thus while diamond does most of its polishing imbedded to 80-90% of its depth in pitch, alumina rolls between lap and part, scoring irregularly and heavily. Consistent with this view, 0.3 micrometer alumina produces surfaces as rough as 2-3 micrometer diamond. The roughness appears to be a function of gap as predicted.

In retrospect, an elastic model is not unreasonable for superpolishing metals. Penetrations are on the order of tens of atom layers. Parts per millions of hardening inclusions are not met in great numbers in such small penetrations. In addition, near the surface there is an entropy gradient with effects analogous to a higher temperature in kinetics normal to a surface. In this region cold work twinning may not have so significant an effect in producing slip plane discontinuities. The surface may be in a continuously annealed state. Thus in a region a few atom layers thick, shearing may involve rupturing bonds whose strength is measured by elasticity. These observations may also apply to tool shearing at cutting depths encountered in the final machining.

Summary

Precision machining is today an important increment to the optical fabrication repertoir; adding much, replacing little. Through a historical survey and a discussion of machine design, we have tried to give a feel to the pace of development and the degree to which further developments may be anticipated. Metrology, for example, has advanced to the level of a few atom layers. Further dramatic increases in capability from metrology precision are unlikely, but developments from more sophisticated uses of metrology and servo-technology are surely possible. Machine design, environmental control, and numerical control are being applied with increasing understanding and sophistication. Tooling and machinability studies are in their infancy, in many cases waiting for third generation machines for sufficient control to sort out effects. Polishing techniques are in a similarly primitive state.

Finally, we have attempted to point out that precision machining does not stand alone as a fabrication process, that successful applications will stem from the cooperative planning of optical designers, machine technologists, and optical finishers.

Acknowledgments

The third generation machines discussed in this paper are just now coming into operation. All the optical turning experience at LLNL is based upon parts turned on DTM-1 and DTM-2, first generation machines by J. Bryan and his group who are now engaged in bringing DTM-3, a third generation machine on line. Their input and cooperation is greatly appreciated.

"Work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract W-7405-ENG-48."

References

1. Yoshioka, J., Koizumi, K., Shimizu, M., Yoshikawa, H., Miyashita, M., Kanai, A., "Surface Grinding With a Newly Developed Ultra Precision Grinding Machine," SME, Precision Machining Workshop, MR82-930, June 8-10, 1982.

2. Bryan, J. B., Brewer, W., McClure, E. R., Pearson, J. W., "Thermal Effects in Dimensional Metrology," Mechanical Engineering, (Feb. 1966).

3. Bryan, J. B., Bowerbank Jr., J. E., Holland, E. D., Mohl, O., "Upgrading Tracer Lathe Machining Operations," ASME, Paper No. 362, Vol. 61. b.1 (1961).

4. Bryan, J. B., Pearson, J. W., "Machine Tool Metrology," ASM/ASME, 1068-753, prepared for WESTEC North Engineering Conference and Exposition (1968).

5. Bryan, J., Clouser, R., Holland, E., "Spindle Accuracy," American Machinist, (Dec. 1967).

6. Donaldson, R. R., "A Simple Method for Separating Spindle Error From Test Ball Roundness Error," Ann. CIRP, V.21/1 (1972).

7. McClure, E. R., "Significance of Thermal Effects in Manufacturing and Metrology," CIRP, Vol XV, pp. 61-66, (1967).

8. Bryan, J. B., "International Status of Thermal Error Research," CIRP, Vol XVI, pp. 203-215, (1968).

9. Bryan, J. B., Clouser, R. W., McClure, R. E., "Expansion of a Cutting Tool During Chip Removal," CIRP, Vol XVI, pp. 49-51, (1968).

10. Watts, R. G., McClure, E. R., "Thermal Expansion of the Workpiece During Turning," ASME 68-WA/PROD-24 (1968).

11. McClure, E. R., Thal-Larsen, H., "Thermal Effects in Precision Machining," ASME 70-WA/Prod-25 (1970).

12. Collicott, H. E., Hartter, L. L., "Thermal Effects in the Manufacturing Process," presented at the National Bureau of Standards Conference on Dimensional Accuracy in Manufacturing (1972).

13. Bryan, J. B., Donaldson, R. R., Clouser, R. W., Blewett, W. H. "Reduction of Machine Tool Spindle Growth," UCRL-74672, (March 1973).

14. McClure, E. R., Bryan, J. B., "Thermal Stability-Key to NC Accuracy," UCRL-72231, 7th Annual Numerical Control Society Proceedings, (Dec. 1969).

15. Bryan, J. B., Donaldson, R. R., and McClure, E. R., "A Practical Solution to the Thermal Stability Problem in Machine Tools," SME Tech. Paper MR72-138 (1972)

16. Bryan, J. B., Donaldson, R. R., McClure, E. R., Whelan, H. A., and Clouser, R. W., "Diamond Turning of Parabolic Mirrors," SPIE Vol 39 (1973)

17. Pearson, J. W., "Precision Machining Commercialization," SPIE Vol. 159 (1978).

18. Saito, Theodore T., "Diamond-Turning of Optics is Stimulating Other Applications or 'One Good Turn Deserves Another'," SPIE Vol. 159 (1978)

19. Johnson, F. E., "Diamond Turning at Honeywell," SPIE Vol. 159 (1978)

20. Johnson, F. E., "Applications of Diamond Turning to Infrared Optical Systems," SPIE Vol. 93 (1976).

21. Brehm, P. D., "Diamond Machining of Metal & Plastic Optics," SPIE Vol. 93 (1976).

22. Johnson, A., "Capabilities to Manufacture Precision Equipment" SPIE Vol. 93 (1976).

23. Bryan, J. B., "Design and Construction of an Ultra Precision 84 Inch Diamond Turning Machine," Precision Engineering, 1(1) (Jan. 1979)

24. Donaldson, R. R., Patterson, S. R., Thompson, D. C., "Diamond Machining and Mechanical Inspection of Optical Components," SPIE Vol. 316 (1981).

25. Decker, D. L., Bennett, J. M., Soileau, M. J., Porteus, J. O. Bennett, H. E., "Surface and Optical Studies of Diamond Turned and Other Metal Mirrors," Proc. SPIE, Vol. 93 (Aug. 1976).

26. Pearson, J. W., "Precision Machining vs Optical Process," SPIE Vol. 163, (April 1979).

27. Decker, D. L., and Grandjean, D. O., "Physical and Optical Properties of Surfaces Generated by Diamond-Turning on an Advanced Machine," Laser Induced Damage in Optical Materials, ed. by A. J. Glass and A. H. Guenther, Washington, D. C. NBS Spec. Publ. 541 (Dec 1978).

28. Arnold, J. B., Saito, T. T., Sladky, R. E., Steger, P. J. Woodall, N. D., "Machining Nonconventional-Shaped Optics," SPIE Vol. 93 (1976)

29. Boyle, M. J. and Ahlstrom, H. G., "Imaging Characteristics of an Axisymmetric, Grazing Incidence X-Ray Microscope Designed for Laser Fusion Research," Rev Sc. Instrum., 49(6) (June 1978).

30. Lampton, M., Cash, W., Malina, R. F., and Bowyer, S., "Design, Fabrication and Performance of Two Grazing Incidence Telescopes for Celestial Extreme Ultraviolet Astronomy," Proc. SPIE, Vol. 106 (April 1977).

31. Ferguson, R. T., Gutheinz, L. M., Mayo III, J. W., German, J. D., Lowrey, W. H., Konopnicki, M. J., "Conical Element Nomenclature, Use, and Metrology," Optical Engineering, Vol. 21, No. 6 (Nov.-Dec. 1982).

32. McKeown, P. A., Wills-Moren, W. J., Read, R. F. J., and Modjarrad, H., "The Design and Development of a Large Ultra Precision CNC Diamond Turning Machine," SME, Precision Machining Workshop, MR82-931 (June 8-10, 1982).

33. Thompson, D. C., Chrislock, J. L., and Newton, L. E., "Development of an Inexpensive, High-Accuracy Diamond Turning Machine," Precision Engineering, 4(2) (April 1982).

34. Bryan, J., Brooks, H., Lamb, J. and Willett, G., "Influence of Air-Borne Noise on the Structures of Precision Machine Tools and Gaging," SME, MR74-936 (1974).

35. Donaldson, R. R., "Error Budgets," Technology of Machine Tools, Vol. 5, Machine Tool Accuracy, Sec. 9.14, UCRL-52960-5, (1980).

36. Bennett, J. M., Decker, D. L., "Surface Characterization of Diamond-Turned Metal Optics," SPIE Vol. 288 (1981).

37. Stover, J. C., "Surface Characteristics of Machined Optics," SPIE Vol. 93, (1976).

38. Church, E. L., Howells, M. R., Vorburger, T. V., "Spectral Analysis of the Finish of Diamond-Turned Mirror Surfaces," Proc. SPIE 315, 202-218 (1982).

39. Church, E. L., Berry, H. C., "Spectral Analysis of the Finish of Polished Optical Surfaces," presented at 2d International Conference on the Metrology and Properties of Engineering Surfaces, Leicester, England, April 14-16, 1982, Proc. Journal Wear (1982).

40. Church, E. L., Jenkinson, H. A., Zavada, J. M., "Measurement of the Finish of Diamond-Turned Metal Surfaces by Differential Light Scattering," Optical Engineering, Vol. 16, No. 4 (July-Aug. 1977).

41. Church, E. L., and Zavada, J. M., "Residual Surface Roughness of Diamond-Turned Optics," Applied Optics, Vol. 14, No. 8 (Aug. 1975).

42. Church, E. L., Jenkinson, H. A., Zavada, J. M., "Relationship Between Surface Scattering and Microtopographic Features," Optical Engineering, Vol. 18, No. 2 (March-April 1979).

43. Stover, J. C., "Roughness Characterization of Smooth Machined Surfaces by Light Scattering," Applied Optics, Vol. 14, No. 8, (Aug. 1975).

44. Sung, C. C., Friday, W. A., Dickerson, L. L., "Surface Characteristics of Diamond-Turned Mirrors and Their Diffraction Patterns," Optical Engineering, Vol. 21, No. 5 (Sept.-Oct. 1982).

45. Elson, J. M., and Bennett, J. M., "Vector Scattering Theory," Optical Engineering, Vol. 18, No. 2 (March-April 1979).

46. Teague, E. C., Vorburger, T. V., Maystre, D., "Light Scattering from Manufactured Surfaces," CIRP, Vol. 30/2 (1981).

47. Stover, John C., "Roughness Measurement by Light Scattering," Laser-Induced Damager in Optical Materials, 163-168 (1974).

48. Brown, N. J., Baker, P. C., and Parks, R. E., "The Polishing-to- Figuring Transition in Turned Optics," Proc. SPIE Vol. 306 (1982).

49. Bennett, J. M. and Dancy, J. H., "Stylus Profiling Instrument for Measuring Statistical Properties of Smooth Optical Surfaces," Appl. Optics, Vol. 20. No. 10 (15 May 1981)

50. Sommargren, G. E., "Optical Heterodyne Profilometry," Appl. Optics, Vol. 20, No. 4 (15 Feb. 1981).

51. Brown, N. J., Baker, P. C., and Maney, R. T., "The Optical Polishing of Metals," Proc. SPIE Vol. 306 (1982).

Appendix A: Polishing model

Our polishing model assumes spherical particles, fixed to a lap, elastically penetrate a surface, removing a swath equal in cross-section to the cross-section of the penetration as the lap is moved relative to the surface. It is shown in Ref. 1 that this yields

$$\frac{dh}{dt} = \frac{P}{2E}\frac{ds}{dt}$$

where

$\frac{dh}{dt}$ = rate of surface removal

$\frac{ds}{dt}$ = rate of relative displacement

E = Young's modulus

P = pressure.

The number of uniform size particles of diameter D_0 per unit area, h_0, reaches a maximum close packing when the concentration K is unity

$$\eta_0 = \frac{2 \quad K}{\sqrt{3}\, D_0^{\,2}} \, .$$

For a population of uniform size particles this yields a penetration depth h_0 at pressure P

$$h_o = \frac{3}{4} \quad D_o\left(\frac{P}{2KE}\right)^{2/3}$$

While this formula is quite instructive in showing trends in roughness parameter dependence, it is drastically oversimplified and quantitatively meaningless for actual roughness calculations. It predicts penetrations of a fraction of an atom layer for the uniform size population. To obtain a more realistic picture of the actual roughness obtained with a viscoelastic lap medium such as pitch, we need the particulate size distribution and the forces involved. Reference (51) shows that the spherical particulate penetration of such a medium can be given as

$$\Upsilon\ (\zeta) = \frac{F}{4\mu VD} = (\zeta + A\zeta^3 + B\zeta^5)$$

where

F = force
μ = viscosity
V = velocity of penetration
D = partical diameter
ζ = h/D
h = penetration depth.

In terms of the mean number of particles per unit area η_0 and mean diameter D_0, we can using the weight fraction f_i of the "ith" group of a population, describe the number and diameter of that population as

$$\eta_i D_i^{\,3} = f_i\ \eta_o\ D_o^{\,3} \, .$$

Letting

$$\Phi_i\ (\zeta_1, \zeta_2) = \frac{1}{D_i}\int_{\zeta_i}^{\zeta_2} \Upsilon_i(\zeta)\ dh$$

we obtain

$$\bar{F}_i\ (\zeta_1, \zeta_2) = \frac{4\mu D^2}{\Delta t}\,\Phi_i\ (\zeta_1\zeta_2)$$

where

F_i = mean force per particle
Δt = time interval

for the penetration of the lap medium from ζ_1 to ζ_2.

Thus the pitch provides a load cell by which the forces, metal penetrations and wear can be examined over the life of the lap.

Letting $\psi \equiv \phi\ (0,1)$ and $\alpha \equiv \sum_{i=1}^{N} f_i \frac{D_o}{D_i}$

$$H = \frac{4}{\sqrt{3}} \quad \frac{VK\mu\alpha\psi}{E}$$

where

H = mean thickness removal capacity per unit of area
V = mean relative velocity between lap and port.

The roughness becomes a function of time and polishing history. That due to the "ith" group among the number N_c of population size groups in contact with the surface is

$$h_i = h_o \left(\frac{\gamma_i \sqrt{D_i D_0}}{\sum_{i=1}^{Nc} f_i \left(D_o^3 / D_i^2 \right) \gamma_i} \right)^{2/3} .$$

Appendix B: Frequency response of flexible belt laps

Ripple occurs in several modes on single point machined parts; as the fundamental of the periodic feed grooves, and as a sinusoidal periodicity from rollers, warped leadscrews etc. Reference 48 examines the response of the surface to a flexible belt lap given on initial sinusoidal surface ripple of amplitude A_0 and frequency "α".

Given a surface initially

$$y_r = A_0\ (1 - \cos \alpha\ X)$$

it was shown that the pressure function at the belt lap interface is

$$-Pr = P\ (1 + \frac{\zeta A}{\delta(1+\zeta)} \cos \alpha x)$$

where $\zeta \equiv (EI\alpha^4 + T\alpha^2 + b)\ \delta/P$

E = Young's modulus of belt
I = moment of inertia per unit width of belt
T = belt tension per unit width
P = platten pressure
b = platten compressability (force/dimension^3)
ζ = $\sqrt{}$ particle penetration into metal mean

Thus maximum peak pressure p_p and minimum valley pressure p_v are related to the mean pressure p as

$$p_p \equiv P\ (1+\eta A);\ p_v = P\ (1-\eta A)$$

but from the wear equation

$$(Yp-Yv) = KV\ (p_p-p_v) = KVP(-2\eta A) = 2\eta A\frac{dy}{dt} = 2\frac{dA}{dt}$$

$$\text{Thus } A = A_o EXP\ (-\eta y) \approx A_o\ EXP\ \left(\frac{-EI\alpha^4 y}{P}\right)$$

$$\text{where } \eta = \frac{EI\alpha^4 + T\alpha^2 + b}{P + \delta\ (EI\alpha^4 + T\alpha^2 + b)}$$

on the conditions that

$$\alpha^2 < \sqrt{\left(\frac{T}{2EI}\right)^2 + \frac{1}{EI}\left(\frac{P}{A_o - \delta} - b\right)} - \frac{T}{2EI} \quad \text{and } A_o > \delta$$

The effect of a fourth power frequency in the exponential leads to rather abrupt changes in phenomena.